VIBRATIONDATA MATLAB GUI PACKAGES · WORKED EXAMPLES

Launch Vehicle & Spacecraft Loads GUI

Seven worked examples checked against hand calculations: modes, Craig-Bampton, force limiting, random, factors of safety, gust and buffet

vibrationdata_lv_sc_loads_a.mlapp · ver 1.0

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Vibrationdata

ABOUT THE PACKAGE

What the GUI does

One window with 13 tabs for launch vehicle and spacecraft loads: models, modes, coupled loads, load factors, sine/random/force-limited test levels, pogo and ascent airloads.

1

Models and modes

Spring-mass chains or workspace M, K; normal modes, effective mass, Craig-Bampton and LV/SC coupling.

2

Coupled loads and load factors

CLA transient, steady + dynamic combination, NASA-STD-5001A factors, mass-acceleration curve.

3

Test environments

Sine with notching, random (Miles and exact), force-limited random vibration for any number of DOFs.

4

Pogo and ascent airloads

SRM modes, MECO, combustion class; static-elastic trim, NASA discrete gusts and buffet.

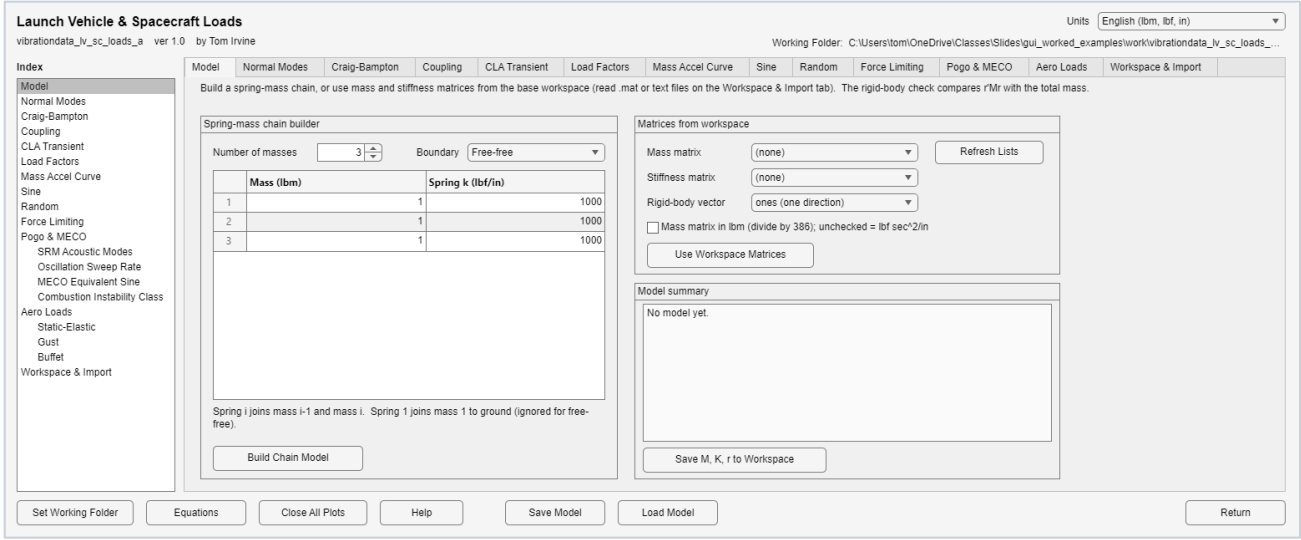
How to open the GUI

1. Staged app (ver 1.0): not yet on the vibrationdata hub or in a release zip

2. In MATLAB: addpath the lv_sc_loads_build folder (holds the app, lvsc_*.m and the equations PDFs)

3. Type vibrationdata_lv_sc_loads_a; pick English or metric units at the top right

4. Use the Index list or the tabs; Equations opens the PDF for the current tab, Return closes the window



Opening window of vibrationdata_lv_sc_loads_a.mlapp (ver 1.0), captured with exportapp in MATLAB R2024a.

Examples in this deck

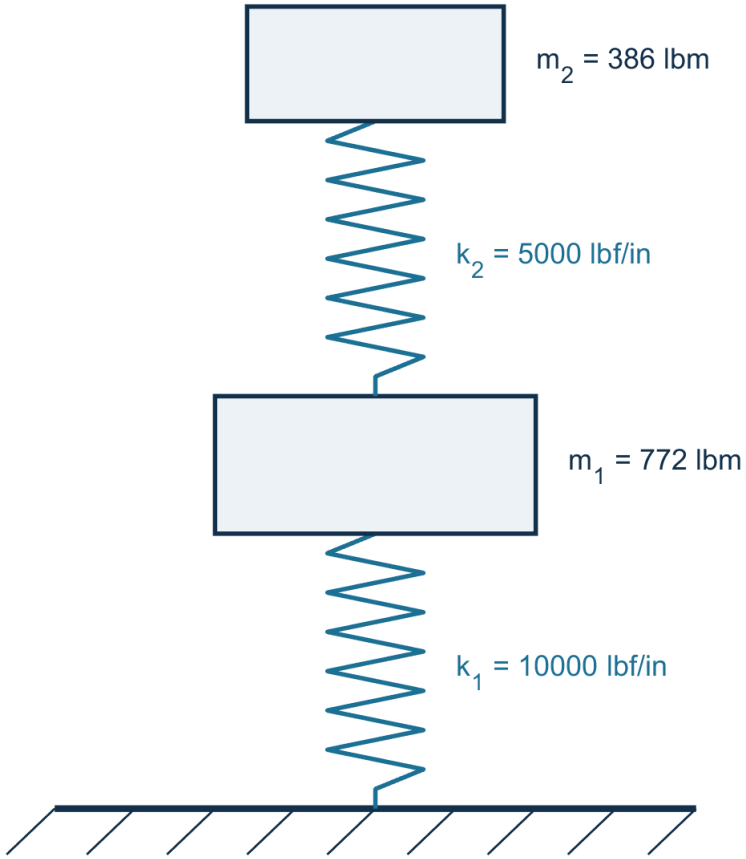
Each example: problem and inputs, the GUI result, and a check against a hand calculation or closed-form solution.

1. **2-DOF normal modes**
   Two-mass chain: frequencies and effective mass
2. **Craig-Bampton reduction**
   Keeping all modes reproduces the full model
3. **Force-limited random test**
   Semi-empirical force limit, 3-DOF demo, C^2 = 2
4. **Miles vs exact SDOF random**
   SDOF random response: Miles equation vs exact
5. **Limit load and factors**
   Limit load, NASA-STD-5001A factors and margins
6. **Gust criterion and trim**
   NASA 1997 discrete gust and rigid gimbal trim
7. **Buffet input formats**
   Buffet: nF(n) and Cn PSD give the same rms

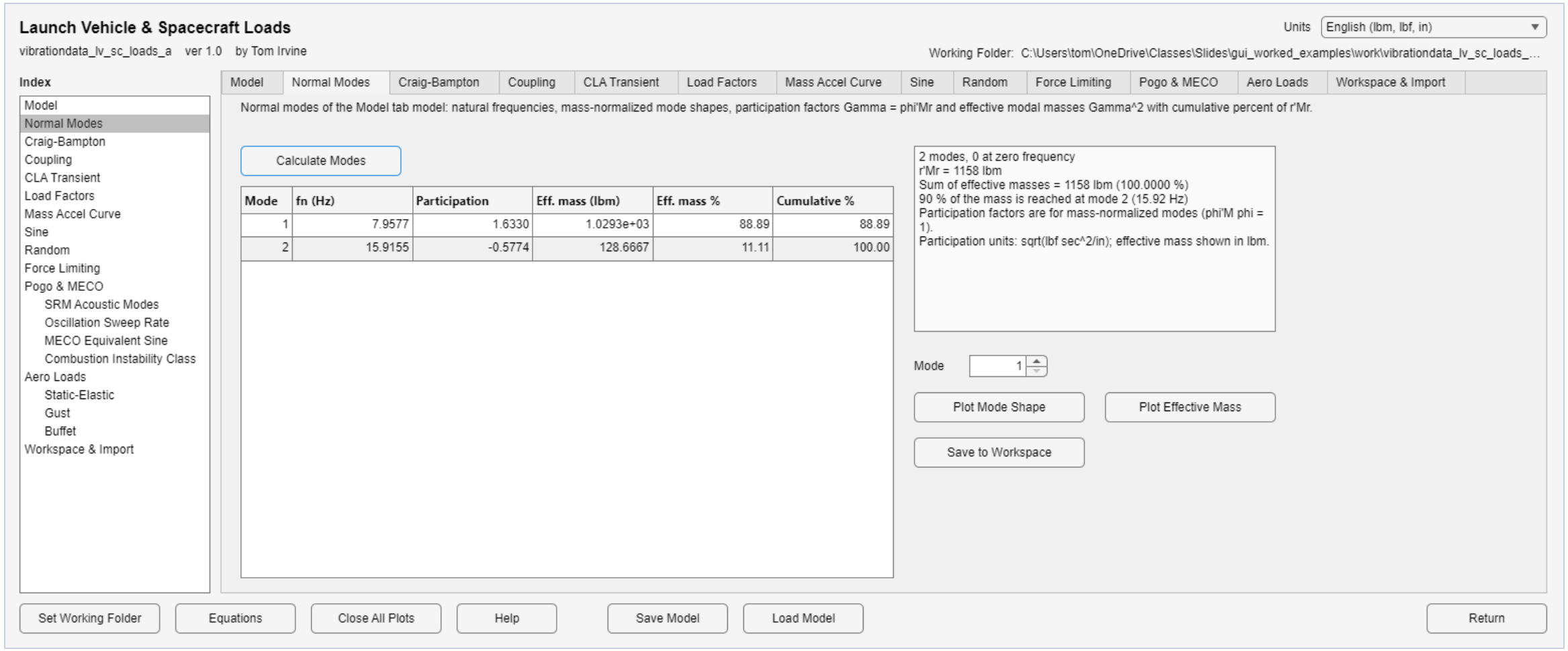
Two-mass chain: frequencies and effective mass

A 772 lbm mass on a 10000 lbf/in spring to ground carries a 386 lbm mass on a 5000 lbf/in spring. Find both natural frequencies and the modal effective masses, and show that the effective masses add up to the total mass.

Input	Value	Units
Mass m1	772	lbm
Mass m2	386	lbm
Spring k1 (ground to m1)	10000	lbf/in
Spring k2 (m1 to m2)	5000	lbf/in
Boundary	Mass 1 to ground	-
Units	English (lbm/386)	-

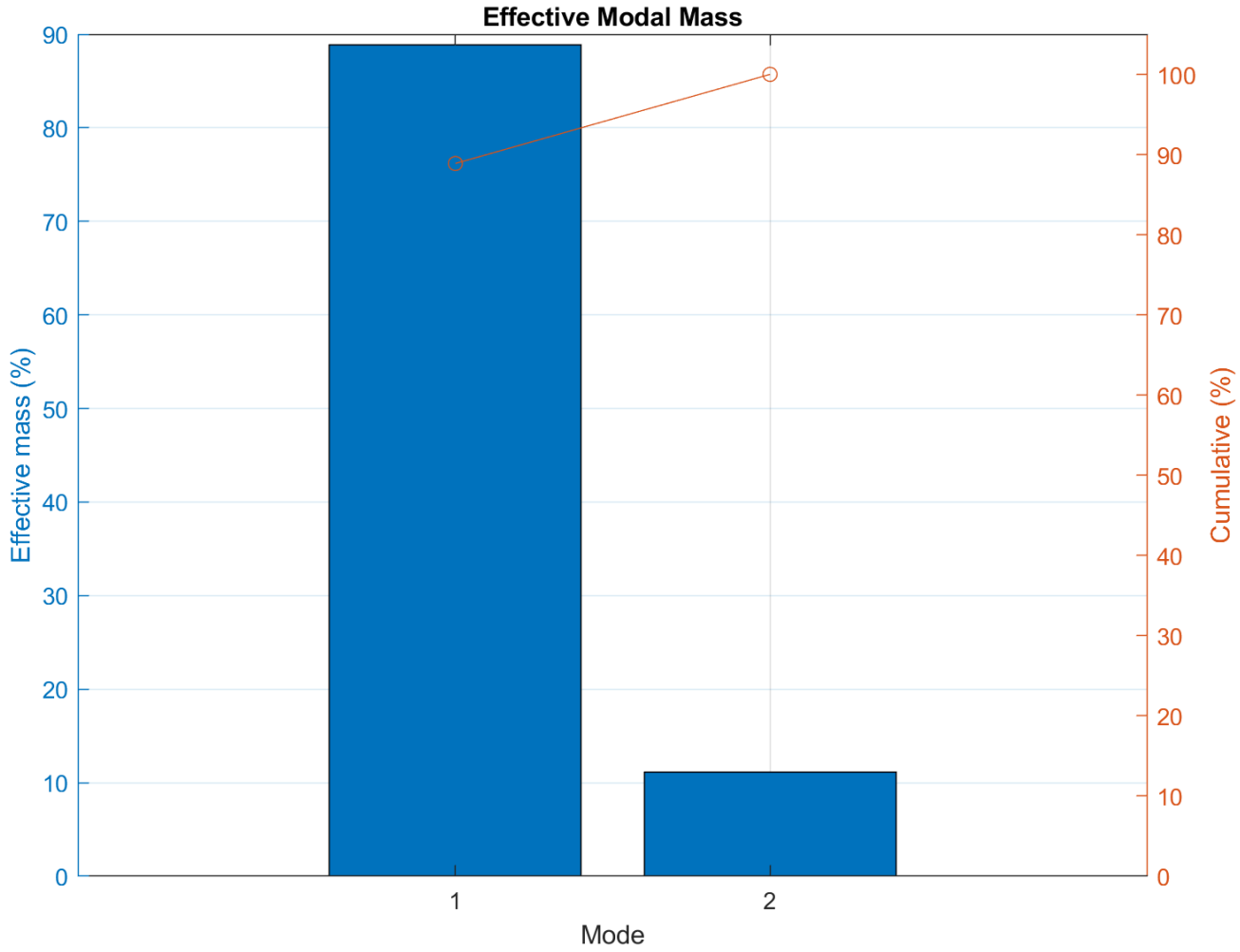


2-DOF normal modes



Model tab: 2-mass chain, Mass 1 to ground, Build Chain Model; Normal Modes tab: Calculate Modes. Table: fn and effective mass.

2-DOF normal modes



Plot Effective Mass: mode 1 carries 88.9 % of the mass, mode 2 the remaining 11.1 % (cumulative 100 %).

Check: 2-DOF normal modes

Quantity	GUI	Hand calc	Diff
f1 (Hz)	7.9577	7.9577	0.00%
f2 (Hz)	15.9155	15.9155	0.00%
Eff. mass mode 1 (lbm)	1029.33	1029.33	0.00%
Eff. mass mode 2 (lbm)	128.667	128.667	0.00%
Sum of eff. masses (lbm)	1158	1158	0.00%

$$\det(\mathbf{K} - \lambda\mathbf{M}) = 0: \quad m_1 m_2 \lambda^2 - [m_1 k_2 + m_2(k_1 + k_2)]\lambda + k_1 k_2 = 0$$

$$\lambda = \omega_n^2 = 2500,\ 10000\ (\text{rad/s})^2 \ \Rightarrow\ f_n = \sqrt{\lambda}/2\pi = 7.958,\ 15.915\ \text{Hz}$$

$$m_{eff,i} = (\phi_i^T \mathbf{M}\mathbf{r})^2/(\phi_i^T \mathbf{M}\phi_i), \quad \phi_1 = [1 \ \ 2]^T, \quad \phi_2 = [1 \ \ -1]^T$$

$$\textstyle\sum_i m_{eff,i} = \mathbf{r}^T\mathbf{M}\mathbf{r} = m_1 + m_2 = 1158\ \text{lbm} \qquad (m \text{ in lbm/386})$$

m1 = 772/386 = 2, m2 = 1 lbf sec^2/in; 2 lam^2 - 25000 lam + 5e7 = 0
lam = 2500 and 10000 -> wn = 50 and 100 rad/sec -> f = 7.958 and 15.915 Hz
Mode 1: phi = [1 2], phi'Mr = 4, phi'M phi = 6 -> meff = 16/6 = 2.667 = 1029.3 lbm
Mode 2: phi = [1 -1], phi'Mr = 1, phi'M phi = 3 -> meff = 1/3 = 128.67 lbm; sum = 1158 lbm

✓ **GUI matches the closed-form 2-DOF solution exactly; effective masses sum to the total mass**

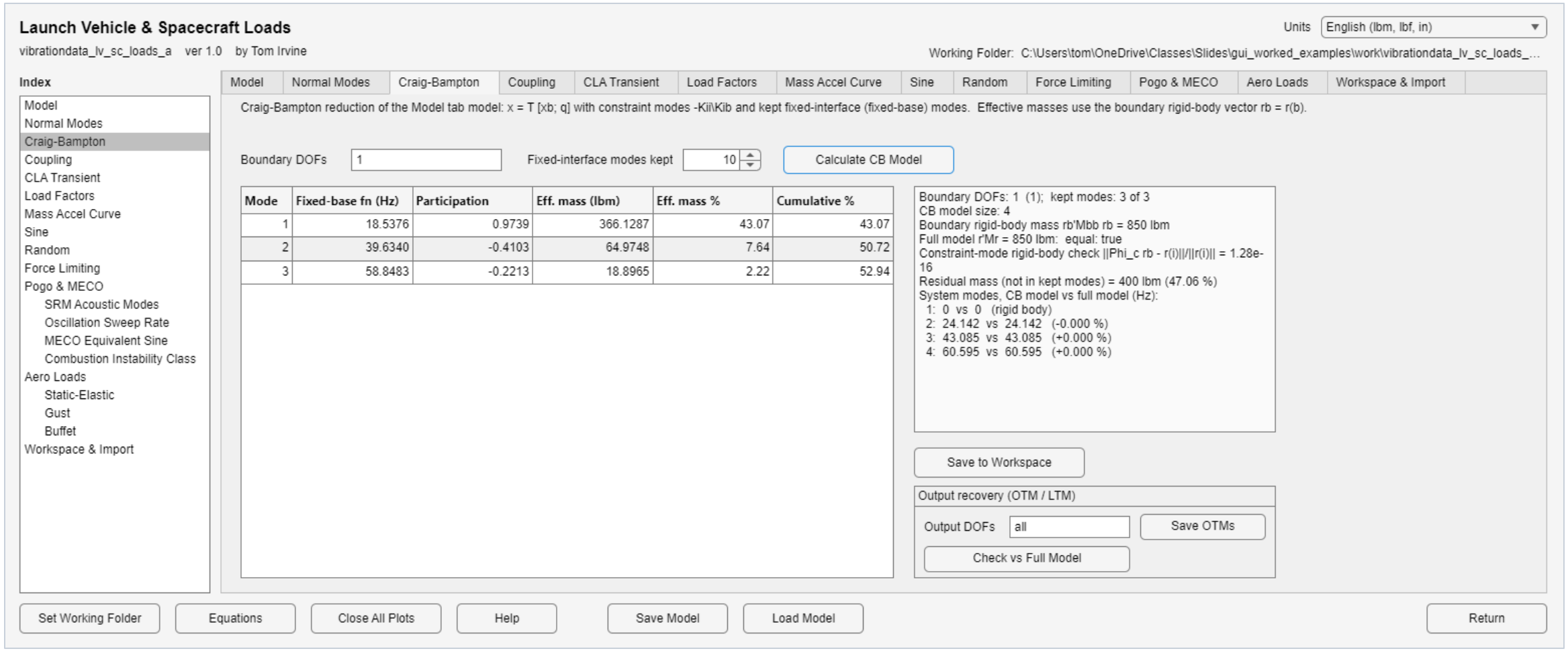
References: W.T. Thomson and M.D. Dahleh, Theory of Vibration with Applications, 5th ed., Prentice Hall, 1998 · vibrationdata_lv_sc_loads_a equations PDF, lvsc_eq_modes.pdf

Keeping all modes reproduces the full model

A free-free 4-mass chain (400, 200, 150, 100 lbm; springs 30000, 20000, 10000 lbf/in) is reduced with DOF 1 as the boundary. With all 3 fixed-interface modes kept, the CB model must give the full-model frequencies; with 1 mode it gives an upper bound.

Input	Value	Units
Masses	400 200 150 100	lbm
Springs	30000 20000 10000	lbf/in
Boundary	Free-free	-
Boundary DOFs	1	-
Fixed-interface modes kept	10 (all 3 used)	-

Craig-Bampton reduction



Craig-Bampton tab after Calculate CB Model: fixed-base modes (table) and CB vs full system modes in the summary.

Check: Craig-Bampton reduction

Quantity	GUI	Hand calc	Diff
System f2 (Hz), CB	24.142	24.1419	0.00%
System f3 (Hz), CB	43.085	43.0849	0.00%
System f4 (Hz), CB	60.595	60.5953	0.00%
Fixed-base f1 (Hz)	18.5376	18.5376	0.00%
Fixed-base f2 (Hz)	39.6340	39.6340	0.00%
Fixed-base f3 (Hz)	58.8483	58.8483	0.00%
f2 with 1 mode kept (Hz)	24.57	24.142 (bound)	+1.77% (expected)

✓ **CB with all modes kept equals the full model to 5 digits; truncation gives an upper bound**

$$\text{Full model}: \ \det(\mathbf{K}-\omega^2\mathbf{M})=0 \quad (4\ \text{DOF},\ \text{free}-\text{free})$$

$$\mathbf{x}=\mathbf{T}\,[\mathbf{x}_b;\ \mathbf{q}], \quad \mathbf{T}=\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{K}_{ii}^{-1}\mathbf{K}_{ib} & \Phi_n \end{bmatrix}$$

$$\text{All fixed}-\text{interface modes kept}: \ \mathbf{T}\ \text{square, nonsingular} \ \Rightarrow\ \text{same eigenvalues}$$

$$\text{Fixed}-\text{base modes}: \ \det(\mathbf{K}_{ii}-\omega^2\mathbf{M}_{ii})=0$$

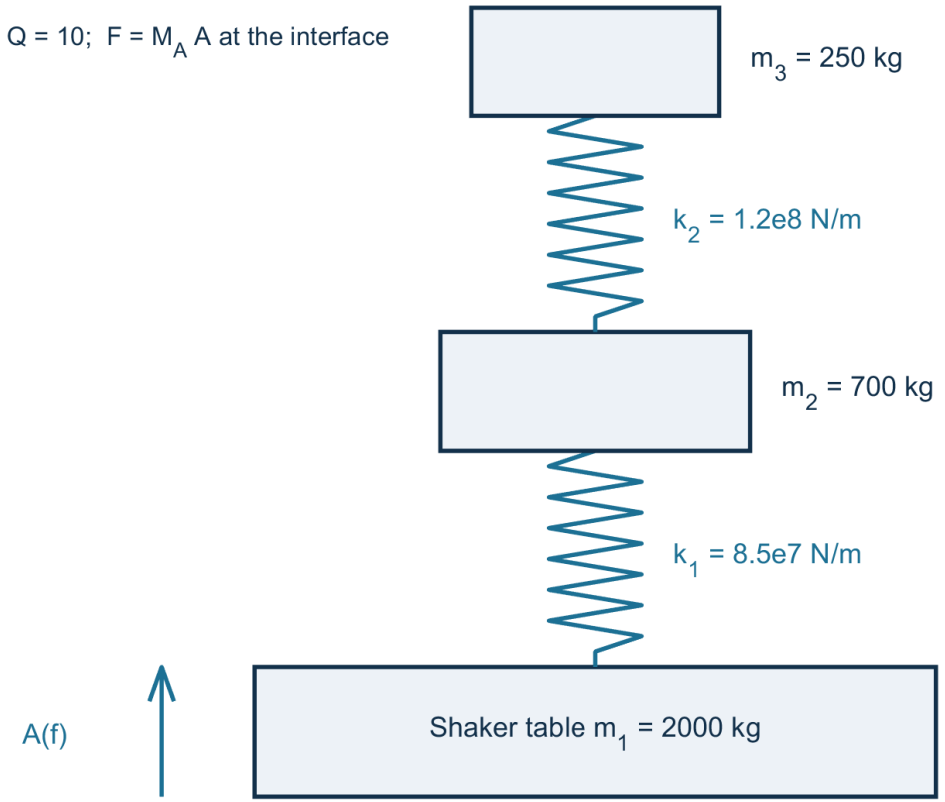
Own 4x4 K and M (lbm/386), eig(K, M): 0, 24.142, 43.085, 60.595 Hz
Ground DOF 1, eig of the 3x3 interior: 18.538, 39.634, 58.848 Hz (fixed-base modes)
All interior modes kept: T is square and nonsingular, so the eigenvalues are unchanged
Truncated to 1 mode: Rayleigh-Ritz gives f2 = 24.57 Hz >= 24.142 Hz, as expected

References: R.R. Craig and M.C.C. Bampton, Coupling of Substructures for Dynamic Analyses, AIAA Journal, 6(7), 1968 · R.R. Craig and A.J. Kurdila, Fundamentals of Structural Dynamics, 2nd ed., Wiley, 2006

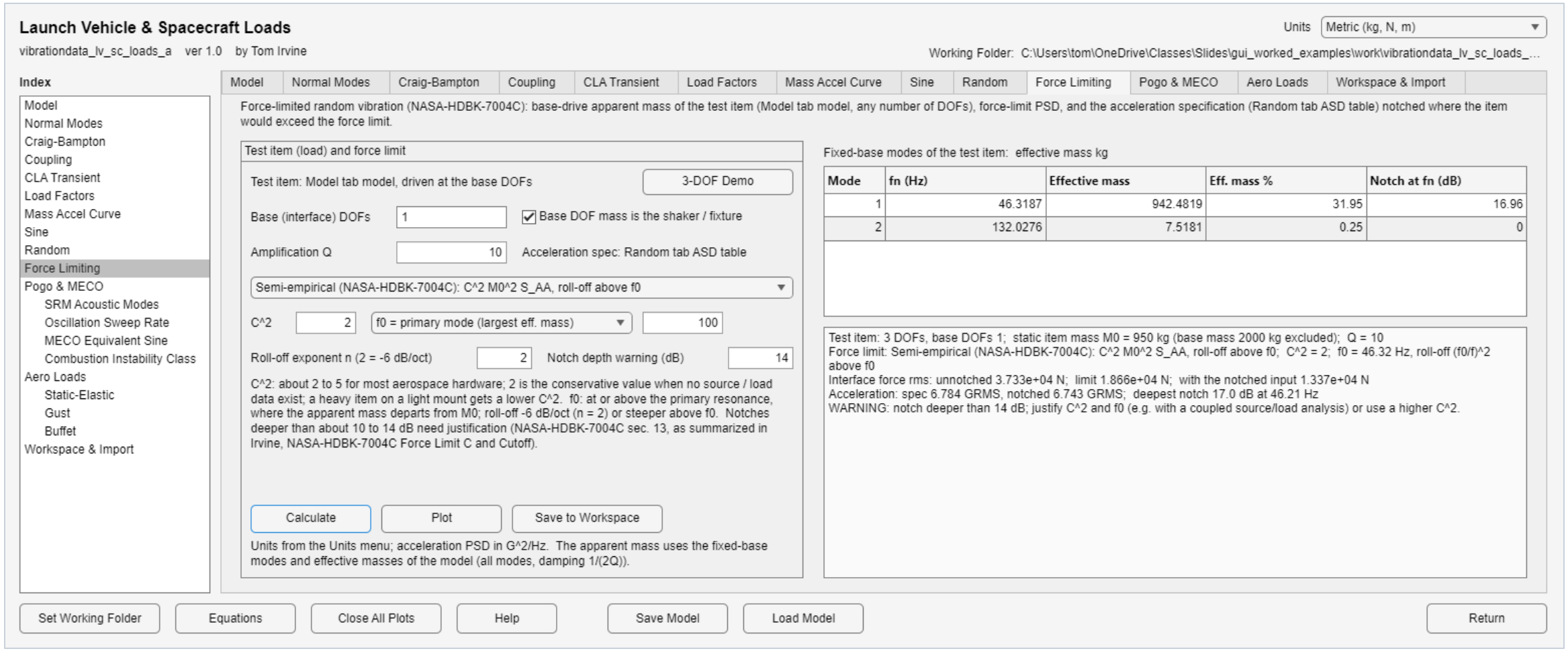
Semi-empirical force limit, 3-DOF demo, C^2 = 2

A 700 kg + 250 kg test item on a 2000 kg shaker table (k1 8.5e7, k2 1.2e8 N/m, Q 10) gets the example random spec. Find the fixed-base modes, M0, the semi-empirical force limit (C^2 = 2) and the notch. The tab works for any number of DOFs.

Input	Value	Units
Shaker table m1 (base DOF 1)	2000	kg
Item m2, m3	700, 250	kg
k1, k2	8.5e7, 1.2e8	N/m
Amplification Q	10	-
C^2 / f0 / roll-off n	2 / primary mode / 2	-
Spec: 20, 80, 500, 2000 Hz	0.01, 0.04, 0.04, 0.01	G^2/Hz
Second case	C^2 = 5, f0 = 60 Hz	-

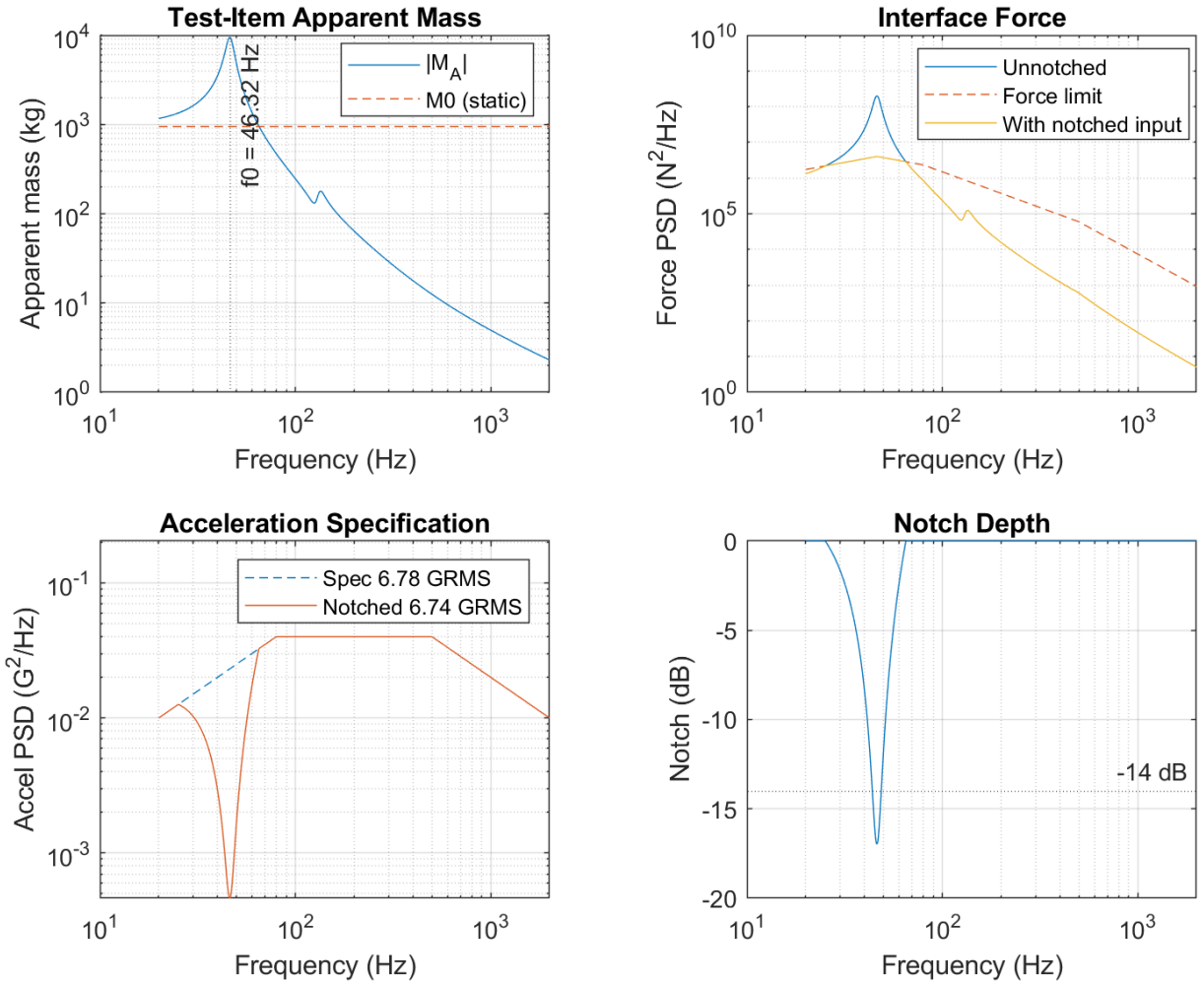


Force-limited random test



Force Limiting tab, metric: 3-DOF Demo, Calculate. Modes table with notch at fn; summary gives M0, force rms and the 14 dB warning.

Force-limited random test



Plot: apparent mass, interface force vs force limit, notched spec (6.74 GRMS) and a 17 dB notch at 46 Hz.

Check: Force-limited random test

Quantity	GUI	Hand calc	Diff
Fixed-base f1 / f2 (Hz)	46.319 / 132.03	46.319 / 132.03	0.00%
M0 (kg) / meff mode 1 (kg)	950 / 942.48	950 / 942.48	0.00%
Deepest notch (dB), C^2 = 2	17.0	16.98	0.12%
Force limit rms (N), C^2 = 2	1.866e4	1.8658e4	0.01%
vs calc_force_limit_psd (N)	1.866e4	1.8621e4	0.21%
Notched spec (GRMS)	6.743	6.7426	0.01%
Notch (dB), C^2 = 5, f0 60	13.0	13.00	0.08%
Force limit rms (N), C^2 = 5	3.588e4	3.5882e4	-0.01%

✓ **GUI matches the hand calc to 0.1 % and calc_force_limit_psd to 0.2 %; 17 dB notch is flagged**

$$M_A(f) = M_0 + \sum_k m_k \frac{\omega^2}{\omega_k^2 - \omega^2 + 2i\zeta\omega_k\omega}, \qquad \zeta = 1/(2Q)$$

$$S_{FF}(f) = C^2 M_0^2\, g^2\, S_{AA}(f) \quad (f \le f_0), \qquad \times (f_0/f)^2 \quad (f > f_0)$$

$$S_{AA,notched} = \min\left[S_{AA},\ S_{FF}/(|M_A|^2 g^2)\right]$$

$$\text{Notch (dB)} = 10\log_{10}\left[|M_A|^2/(C^2 M_0^2)\right] \quad \text{at } f \le f_0$$

Fixed base: eig([2.05e8 -1.2e8; -1.2e8 1.2e8], diag(700, 250)) -> 46.32, 132.03 Hz
meff = (phi'Mr)^2/(phi'M phi) = 942.48 + 7.52 = 950 kg = M0 (shaker mass excluded)
|M_A(46.32 Hz)| = 9473 kg = 9.97 M0; 10 log10(9.97^2/2) = 16.96 dB > 14 dB guideline
C^2 = 5: 10 log10(9.97^2/5) = 12.99 dB; F_lim rms = sqrt(int C^2 M0^2 g^2 S_AA roll df)
Tom's calc_force_limit_psd (C = sqrt 2, mode 3): 1.8621e4 N at g = 9.81; f0 = peak |M_A|

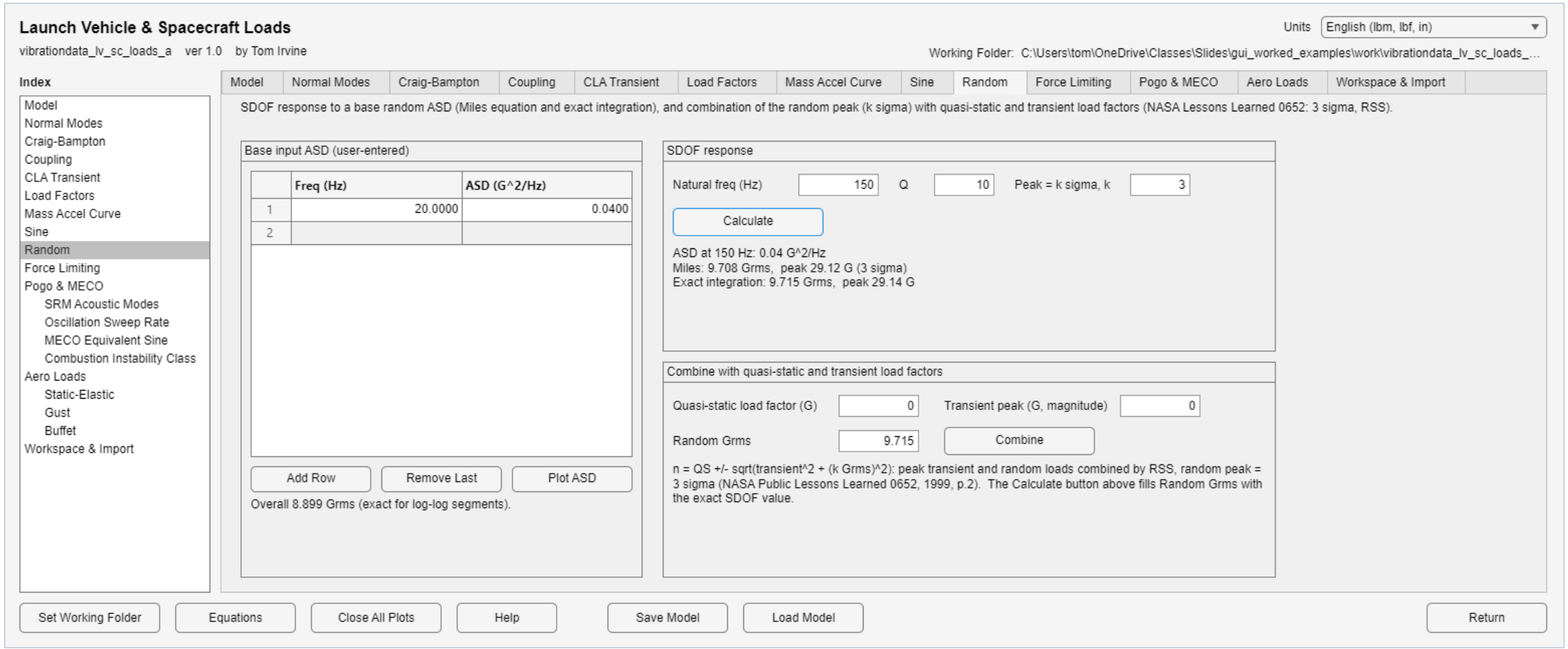
References: T.D. Scharton, NASA-HDBK-7004C, Force Limited Vibration Testing, NASA, 2012 (semi-empirical method) · T.D. Scharton, Force Limited Vibration Testing Monograph, NASA RP-1403, 1997 · vibrationdata_lv_sc_loads_a equations PDF, lvsc_eq_forcelimit.pdf

SDOF random response: Miles equation vs exact

A component with fn = 150 Hz and Q = 10 is mounted on a base with a flat 0.04 G^2/Hz input from 20 to 2000 Hz. Find the response GRMS by Miles equation and by exact integration of the transmissibility, and the 3-sigma peak.

Input	Value	Units
ASD (flat)	0.04	G^2/Hz
Band	20 - 2000	Hz
Natural frequency fn	150	Hz
Q	10	-
Peak factor k	3	sigma

Miles vs exact SDOF random



Random tab: flat ASD table, fn 150 Hz, Q 10, k 3, Calculate. Miles and exact GRMS and peaks; input 8.899 GRMS.

Check: Miles vs exact SDOF random

Quantity	GUI	Hand calc	Diff
Input overall (GRMS)	8.899	8.8994	0.00%
Miles response (GRMS)	9.708	9.7081	0.00%
Exact, 20-2000 Hz (GRMS)	9.715	9.7147	0.00%
3-sigma peak, Miles (G)	29.12	29.12	-0.02%
3-sigma peak, exact (G)	29.14	29.14	-0.01%

$$\text{Miles}: \quad G_{rms} = \sqrt{\tfrac{\pi}{2}\, f_n\, Q\, A}, \qquad A = 0.04\ \text{G}^2/\text{Hz}$$

$$|T(\rho)|^2 = \frac{1+(2\zeta\rho)^2}{(1-\rho^2)^2+(2\zeta\rho)^2}, \qquad \rho = f/f_n, \quad \zeta = 1/(2Q)$$

$$\text{Exact}: \quad G_{rms} = \sqrt{A \int_{20}^{2000} |T|^2\, df}\ ; \quad \text{infinite band} = \text{Miles}\sqrt{1+1/Q^2}$$

Input: sqrt(0.04 x 1980) = 8.899 GRMS
Miles: sqrt(pi/2 x 150 x 10 x 0.04) = sqrt(94.25) = 9.708 GRMS
Exact: MATLAB integral of 0.04 |T|^2 from 20 to 2000 Hz -> 9.7147 GRMS
Infinite band: 9.708 x sqrt(1 + 1/100) = 9.757; the band limits remove 0.4 %

✓ **GUI matches Miles and the exact integral to 0.01 %; Miles is 0.07 % low here**

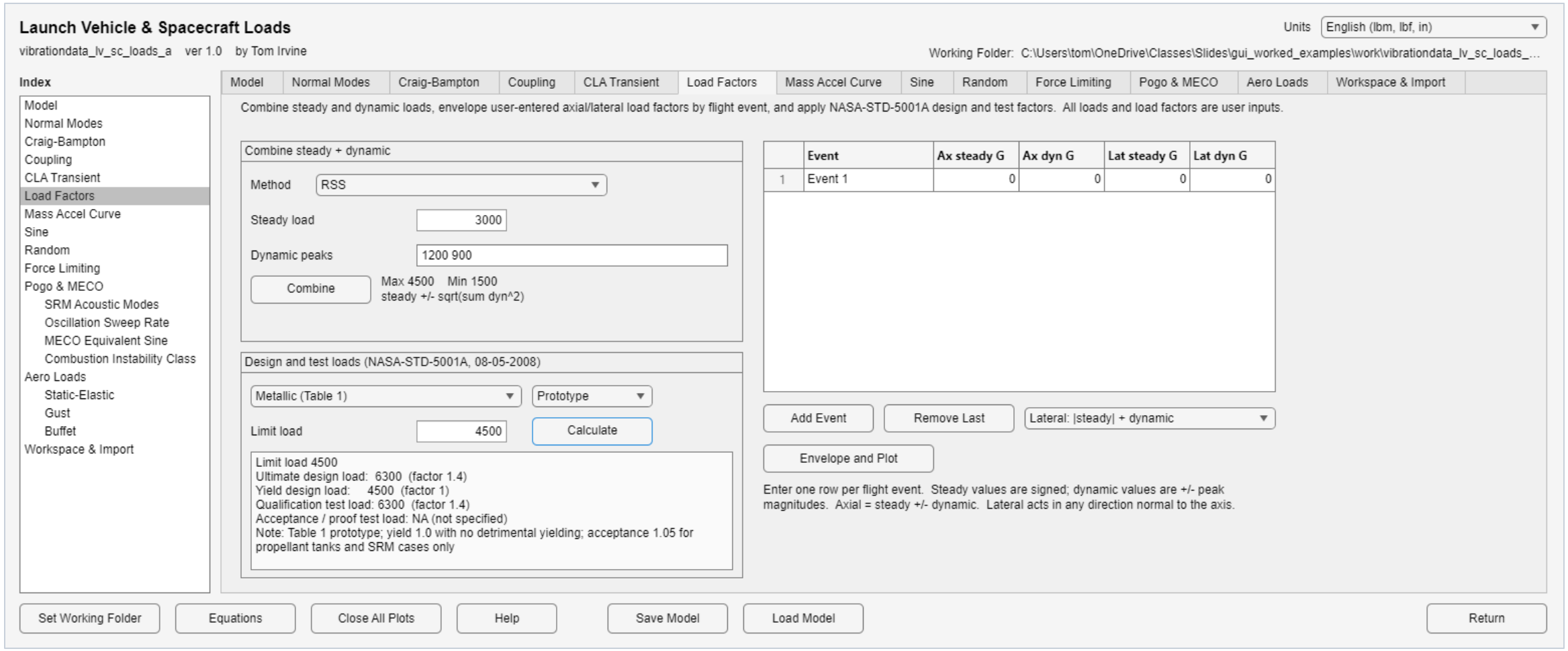
References: J.W. Miles, On Structural Fatigue Under Random Loading, J. Aeronautical Sciences, 21(11), 1954 · Harris' Shock and Vibration Handbook (random vibration chapter) · vibrationdata_lv_sc_loads_a equations PDF, lvsc_eq_random.pdf

Limit load, NASA-STD-5001A factors and margins

A metallic bracket sees a 3000 lbf steady load plus 1200 and 900 lbf dynamic peaks (RSS). Find the limit load and the prototype design and test loads per NASA-STD-5001A, then the margins for 7500 lbf ultimate and 5800 lbf yield allowables.

Input	Value	Units
Method	RSS	-
Steady load	3000	lbf
Dynamic peaks	1200, 900	lbf
Material	Metallic (Table 1)	-
Approach	Prototype	-
Ultimate allowable (hand)	7500	lbf
Yield allowable (hand)	5800	lbf

Limit load and factors



Load Factors tab: RSS Combine fills the limit load; Metallic, Prototype, Calculate gives the design and test loads.

Check: Limit load and factors

Quantity	GUI	Hand calc	Diff
Limit load (lbf)	4500	4500	0.00%
Ultimate design load (lbf)	6300	6300	0.00%
Yield design load (lbf)	4500	4500	0.00%
Qualification test load (lbf)	6300	6300	0.00%
Protoflight yield / accept (lbf)	5625 / 5400	5625 / 5400	0.00%
MS ultimate / yield	- (not in GUI)	+0.19 / +0.29	-

$$P_{limit} = P_{steady} + \sqrt{\sum_i P_{dyn,i}^2}$$

$$P_{ult} = 1.4\,P_{limit}, \quad P_{yield} = 1.0\,P_{limit}, \quad P_{qual} = 1.4\,P_{limit} \quad (\text{metallic, prototype})$$

$$MS = \frac{P_{allowable}}{FS \cdot P_{limit}} - 1 \quad \geq 0$$

Limit = 3000 + sqrt(1200^2 + 900^2) = 3000 + 1500 = 4500 lbf
Prototype metallic: ultimate 1.4 x 4500 = 6300, yield 1.0 x 4500 = 4500, qual 1.4 x 4500 = 6300
Protoflight: yield 1.25 x 4500 = 5625, acceptance 1.2 x 4500 = 5400 lbf
MS_ult = 7500/6300 - 1 = +0.19; MS_yld = 5800/4500 - 1 = +0.29 (both positive)

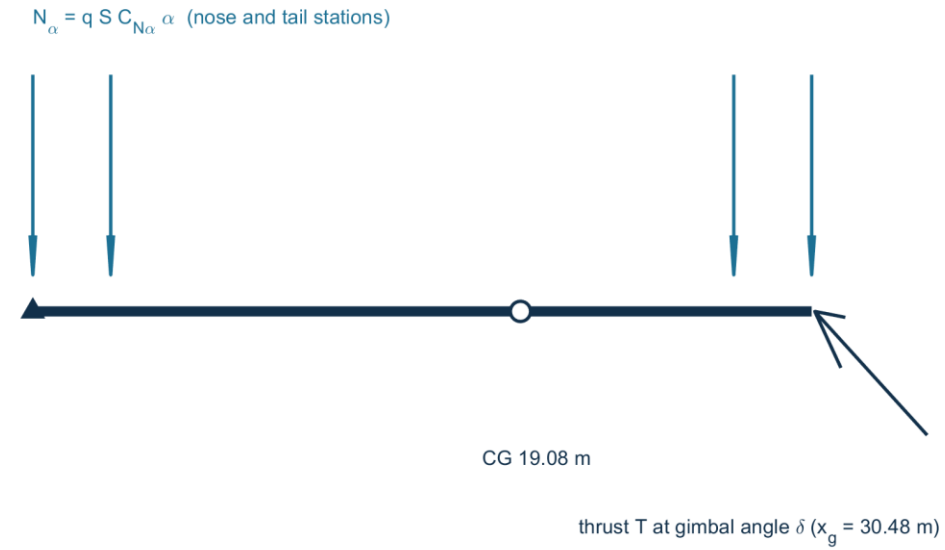
✓ **GUI loads match the hand calc exactly; margins are a hand step (the GUI gives loads, not MS)**

References: NASA-STD-5001A, Structural Design and Test Factors of Safety for Spaceflight Hardware, 2008, Table 1 (the rev the app cites)

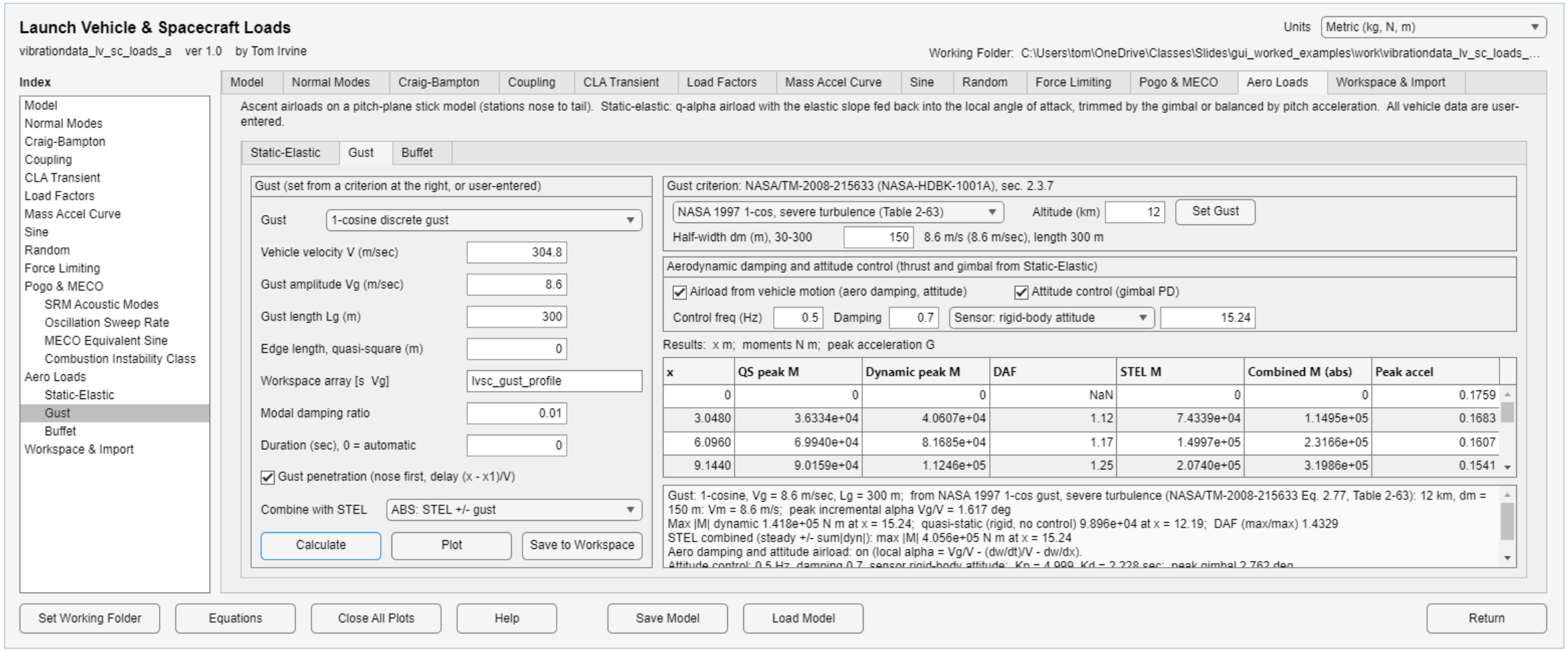
NASA 1997 discrete gust and rigid gimbal trim

Set the design gust for severe turbulence at 12 km with half-width dm = 150 m (and 13 km, 165 m by interpolation, and by Eq. 2.82). For the generic vehicle at q = 34.47 kPa, alpha = 3 deg, check the rigid trim gimbal angle by moment balance.

Input	Value	Units
Criterion	NASA 1997 1-cos, Table 2-63	-
Altitude / dm	12 / 150	km / m
Interpolated case	13 / 165	km / m
q / alpha	34474 / 3	Pa / deg
Ref. area S	7.2966	m^2
Thrust T / gimbal x_g	444822 / 30.48	N / m
Stations	Generic Example (11, metric)	-

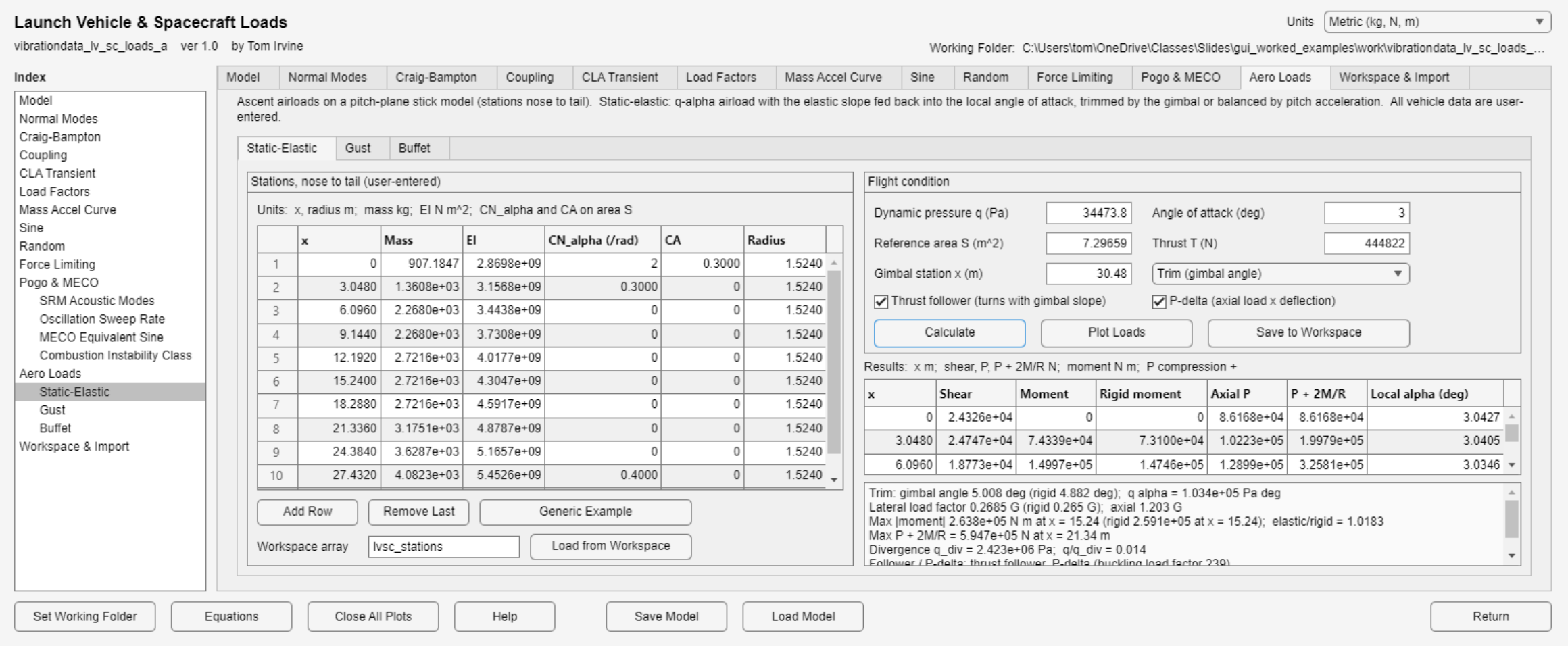


Gust criterion and trim



Gust sub-tab, metric: criterion 12 km, dm 150 m, Set Gust -> 8.6 m/s, 300 m; Calculate gives the gust loads.

Gust criterion and trim



Static-Elastic sub-tab, metric Generic Example, Trim: summary gives gimbal angle 5.008 deg (rigid 4.882 deg).

Check: Gust criterion and trim

Quantity	GUI	Hand calc	Diff
Vm, 12 km, dm 150 m (m/s)	8.6	8.60	0.00%
Gust length 2 dm (m)	300	300	0.00%
Vm, 13 km, 165 m, bilinear (m/s)	7.673	7.6725	0.01%
Vm, Eq. 2.82, 12 km 150 m (m/s)	8.919	8.9195	-0.01%
Rigid trim gimbal angle (deg)	4.882	4.8823	-0.01%
Rigid lateral load factor (G)	0.265	0.2649	0.05%

$$V_m(h, d_m): \text{ bilinear in Table } 2{-}63; \quad L_g = 2\,d_m$$

$$\text{Eq. } 2.82: \quad V_m = \sigma_h \cdot 2.8\sqrt{1 - \tfrac{2}{\pi}\tan^{-1}\tfrac{1}{\pi d_m / L_h}}$$

$$\text{Trim}: \quad \textstyle\sum_i N_{\alpha,i}(x_i - x_{cg}) + T\,\delta\,(x_g - x_{cg}) = 0, \quad N_{\alpha,i} = q\,S\,C_{N\alpha,i}\,\alpha$$

Table 2-63, 12 km: 8.60 (150 m), 9.38 (180 m); 14 km: 6.07, 6.64 -> 8.99, 6.355 -> 7.6725 m/s
Eq. 2.82, sigma 7.89 m/s, L 1800 m: pi dm/L = 0.2618, ratio 1.1305 -> Vm = 8.919 m/s
m = 31298 kg, x_cg = 19.083 m; N_alpha = qS C_Nalpha alpha at 4 stations, sum 43463 N
M_aero(cg) = -4.3199e5 N m; delta = 4.3199e5/(444822 x 11.397) = 0.08521 rad = 4.882 deg

✓ **Gust criterion and rigid trim match the hand calc to 0.01 %; elastic trim (5.008 deg) not hand-checked**

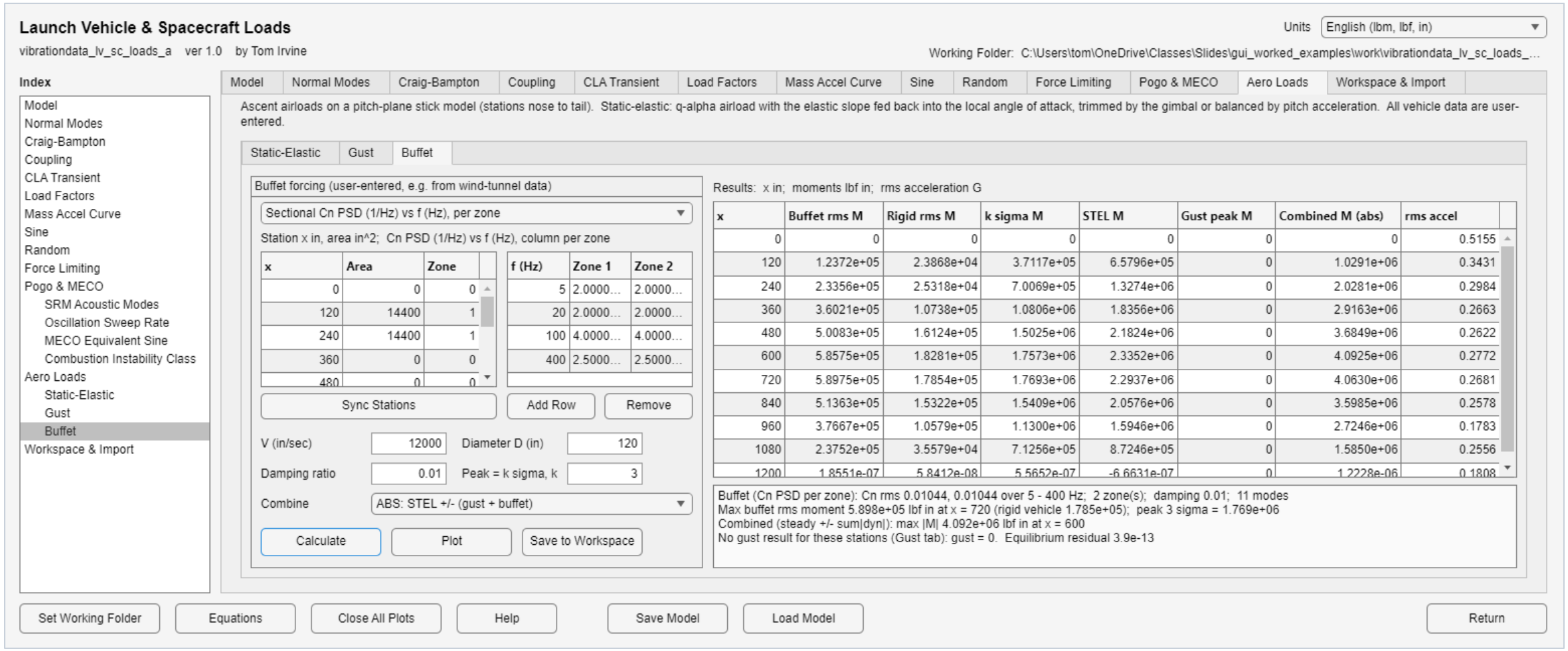
References: NASA/TM-2008-215633, Terrestrial Environment (Climatic) Criteria Guidelines for Use in Aerospace Vehicle Development, 2008 Rev., sec. 2.3.7, Table 2-63 · vibrationdata_lv_sc_loads_a equations PDF, lvsc_eq_aero.pdf

Buffet: nF(n) and Cn PSD give the same rms

The generic vehicle has buffet areas near the nose (zone 1) and tail (zone 2), V = 12000 in/sec, D = 120 in. Enter the normalized spectrum nF(n), then the equivalent sectional Cn PSD. Check the Cn rms in closed form and that both formats agree.

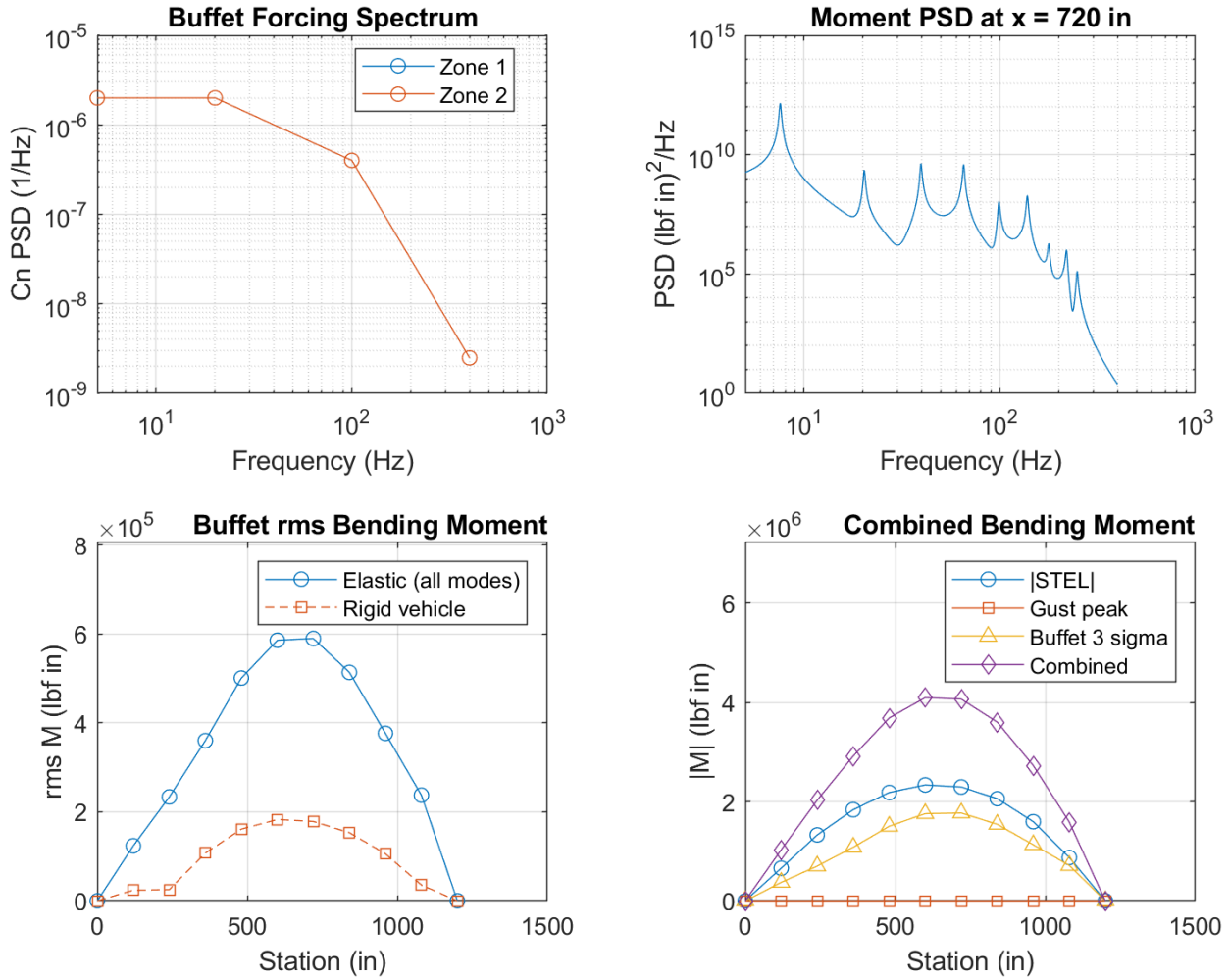
Input	Value	Units
nF(n) at n = 0.05, 0.2, 1, 4	1e-5, 4e-5, 4e-5, 1e-6	-
Velocity V	12000	in/sec
Diameter D	120	in
Buffet area (4 stations)	14400	in^2
Damping ratio	0.01	-
Stations	Static-Elastic Generic Example	-

Buffet input formats



Buffet sub-tab, PSD format (filled from nF(n)), Calculate: Cn rms 0.01044 per zone and the rms moments by station.

Buffet input formats



Plot: Cn PSD per zone (zones overlap), moment PSD at x = 720 in, buffet rms moment and the combined moment.

Check: Buffet input formats

Quantity	GUI	Hand calc	Diff
Cn rms, 5 - 400 Hz	0.01044	0.010442	-0.02%
PSD at 5 / 20 Hz (1/Hz)	2.0e-6 / 2.0e-6	2.0e-6 / 2.0e-6	0.00%
PSD at 100 / 400 Hz (1/Hz)	4.0e-7 / 2.5e-9	4.0e-7 / 2.5e-9	0.00%
Band n V/D (Hz)	5 - 400	5 - 400	0.00%
Max rms M (lbf in): nF vs PSD	5.898e5	5.898e5 (GUI nF)	0.00%

$$\Phi_{C_n}(f) = nF(n)/f, \qquad n = f\,D/V$$

$$C^2_{n,rms} = \int \Phi_{C_n}\, df = \int nF(n)\,\frac{dn}{n}$$

$$\text{Segment } nF = G_1(n/n_1)^b: \quad \int_{n_1}^{n_2} = \frac{G_1}{b}\left[(n_2/n_1)^b - 1\right] \quad (b = 0:\ G_1 \ln \tfrac{n_2}{n_1})$$

f = n V/D = 100 n: band 5 - 400 Hz; Phi = nF/f: 1e-5/5 = 2e-6, 4e-5/20 = 2e-6, 4e-5/100 = 4e-7
Segment 1 (b = 1): 1e-5/0.05 x 0.15 = 3.0e-5; segment 2 (b = 0): 4e-5 ln 5 = 6.44e-5
Segment 3 (b = -2.661): 4e-5/(-2.661) x (4^-2.661 - 1) = 1.466e-5
Sum 1.0904e-4 -> Cn rms = 0.010442; rms moments identical in both formats (all 11 stations)

✓ **Cn rms matches the closed form to 0.02 %; both input formats give identical rms moments**

References: vibrationdata_lv_sc_loads_a equations PDF, lvsc_eq_aero.pdf (buffet section) · Harris' Shock and Vibration Handbook (random vibration chapter)

Summary of worked examples

Example	Check result
1. 2-DOF normal modes	GUI matches the closed-form 2-DOF solution exactly; effective masses sum to the total mass
2. Craig-Bampton reduction	CB with all modes kept equals the full model to 5 digits; truncation gives an upper bound
3. Force-limited random test	GUI matches the hand calc to 0.1 % and calc_force_limit_psd to 0.2 %; 17 dB notch is flagged
4. Miles vs exact SDOF random	GUI matches Miles and the exact integral to 0.01 %; Miles is 0.07 % low here
5. Limit load and factors	GUI loads match the hand calc exactly; margins are a hand step (the GUI gives loads, not MS)
6. Gust criterion and trim	Gust criterion and rigid trim match the hand calc to 0.01 %; elastic trim (5.008 deg) not hand-checked
7. Buffet input formats	Cn rms matches the closed form to 0.02 %; both input formats give identical rms moments

- All 7 examples match independent hand calculations to 0.2 % or better (most to 0.01 %)
- Craig-Bampton with all modes kept equals the full model; truncation gives upper-bound frequencies
- Force limiting matches the hand calc and Tom's calc_force_limit_psd; notches over 14 dB are flagged
- Hand steps beyond the GUI: margins of safety (Ex. 5) and the rigid trim moment balance (Ex. 6)

Examples run in MATLAB R2024a through the GUI (vibrationdata_lv_sc_loads_a.mlapp, ver 1.0). Hand calculations were written independently of the app code.