True Point Impedance Function Summary

This MATLAB function computes the true mechanical point impedance of a finite plate undergoing bending vibration. It implements the theoretical expression for a point excitation on a thin plate below and above the critical (coincidence) frequency, including a smooth blend to the Cremer–Heckl high-frequency constant impedance.

Function Overview

Function: true_point_impedance(fc, E, nu, h, rho, a, eta, r0, p, c0)
Outputs: Zp (impedance), Yp (mobility), k (bending wavenumber), D (bending stiffness), mu (mass per unit area), fc_crit (critical frequency).

Key Physics and Equations

1. The bending stiffness is computed as D = E*h^3 / [12*(1 - nu^2)].
2. The mass per unit area is mu = rho*h.
3. The bending wavenumber is k = ((omega^2 * mu / D)^(1/4)).
4. The true point impedance uses the logarithmic form:
   Z_true = (8*pi*D*k^2) / (j*omega*(1 + j*(2/pi)*log(k*a))).
5. The Cremer–Heckl high-frequency limit is Z_CH = 8*sqrt(D*mu)*(1 + j).
6. The critical (coincidence) frequency in air is fc_crit = (c0^2 / (2*pi))*sqrt(mu/D).

Smooth Blending

A smooth transition between the true point impedance and the Cremer–Heckl constant is applied around the critical frequency. This prevents an abrupt change and models the realistic shift between sub-critical and super-critical regimes:

r = fc / fc_crit
w = 1 / (1 + (r/r0)^p)
Zp = w * Z_true + (1 - w) * Z_CH.

Parameter 'a' (Effective Contact Radius)

The parameter 'a' represents the effective contact radius of the applied force or measurement point. It regularizes the mathematical singularity of a perfect point load and depends on the physical contact size.
Typical values:

- Hammer or shaker tip: 2–5 mm radius
- Accelerometer base: ~1 mm radius
- Numerical or very small contact: 0.2–0.5 mm radius

Smaller 'a' results in a more pronounced impedance decrease below the critical frequency.

Expected Impedance Behavior

The function reproduces the following mechanical impedance behavior for thin plates:
• Below critical frequency (f < fc): |Zp| ∝ f^0.5 — the plate is compliant and impedance decreases at low f.

• Near fc: transition or dip in impedance due to wave speed matching with the surrounding air.
• Above fc: |Zp| tends toward the constant Cremer–Heckl plateau.

Typical Interpretation

A correctly tuned impedance plot should show:
1. Rising impedance at low frequencies (f^0.5 trend).
2. A gentle dip or transition near the critical frequency (~1–2 kHz for 6.25 mm aluminum).
3. A flat asymptotic region at higher frequencies representing the Cremer–Heckl limit.

Validation

The user's impedance plot shows the correct qualitative behavior:
• Increasing impedance below 1 kHz (bending regime).
• A mild dip near the predicted critical frequency (~1.9 kHz).
• Flattening above 3–5 kHz toward the expected plateau.

This confirms that the implemented equations and blending logic are consistent with theory.

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Frequency Regime Summary

The table below summarizes the expected qualitative behavior of the real and imaginary parts of the mechanical point impedance across different frequency regions relative to the critical (coincidence) frequency.

Frequency Region	Dominant Physics	Imag(Z_p) Behavior
Low (f ≪ f_c)	Bending stiffness dominates	Imag(Z_p) negative (compliant response)
Around f_c	Wave speed ≈ sound speed (transition region)	Imag(Z_p) crosses zero
High (f ≫ f_c)	Radiation and inertia dominate (C&H regime)	Imag(Z_p) ≈ +Re(Z_p), constant magnitude